OPTIMIZATION OF FERTIGATION PRACTICE

AN APPROACH BASED ON VIABILITY THEORY AND OPTIMAL CONTROL

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OUTLINE

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- **Motivation**
- **Crop fertigation model**
- **Problem formulation**
- **Optimal strategies**
- **Conclusion**

MOTIVATION

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Population growth

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Food demand

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Climate change

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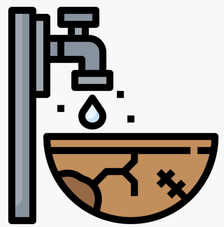

Water scarcity

MOTIVATION

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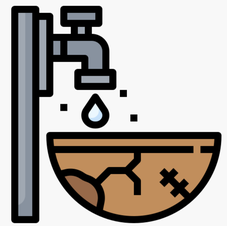

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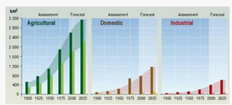

60% of mobilized water for agriculture

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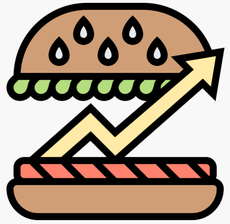

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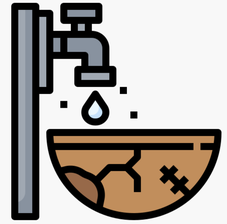

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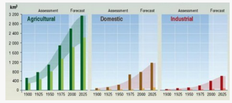

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- Increase agricultural production to meet food needs
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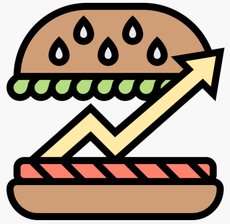

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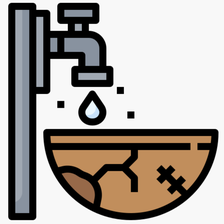

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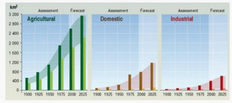

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- Optimize agriculture irrigation with treated wastewater

CROP FERTIGATION MODEL

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- Water balance equation :

$$\dot{S} = k_1\Big(-\varphi(t)K_S(S) - (1-\varphi(t))K_R(S) + k_2 u\Big)$$

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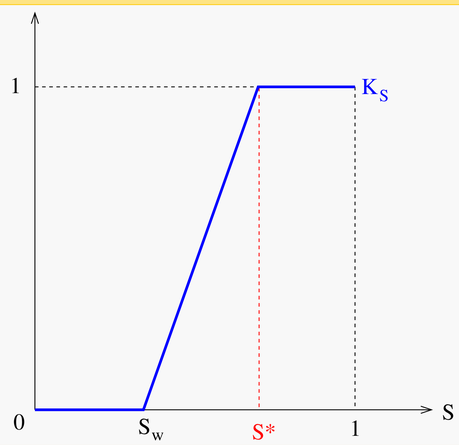

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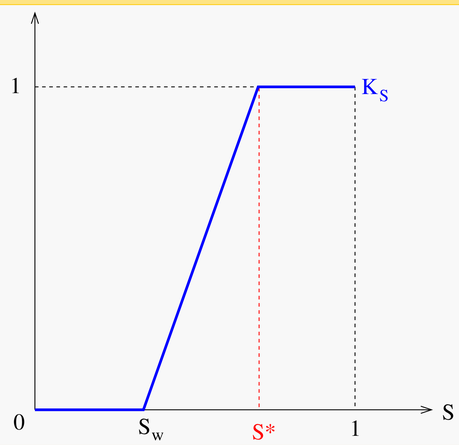

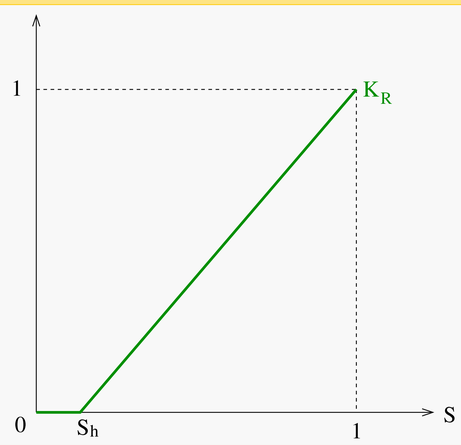

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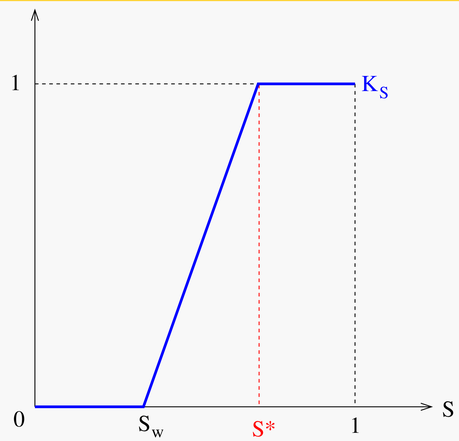

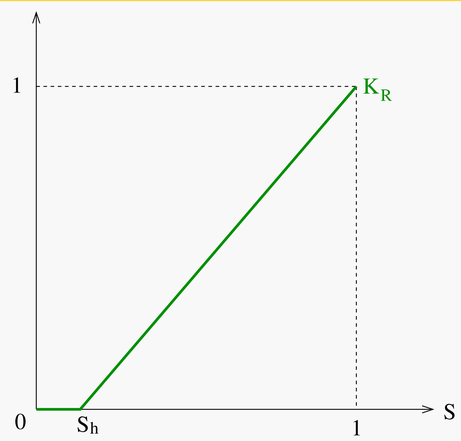

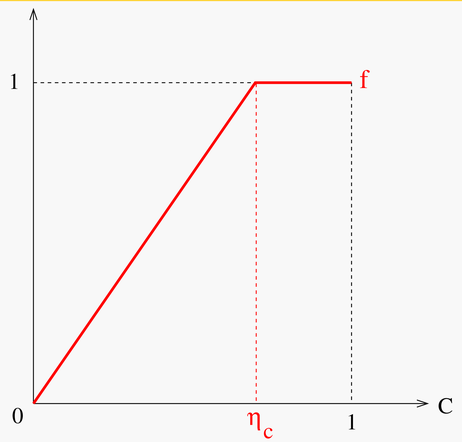

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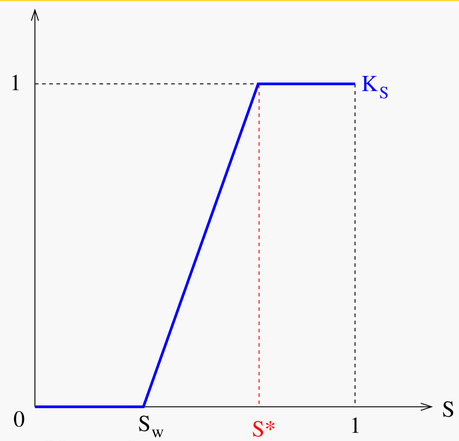

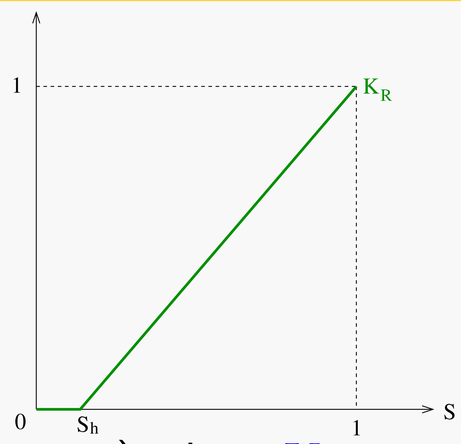

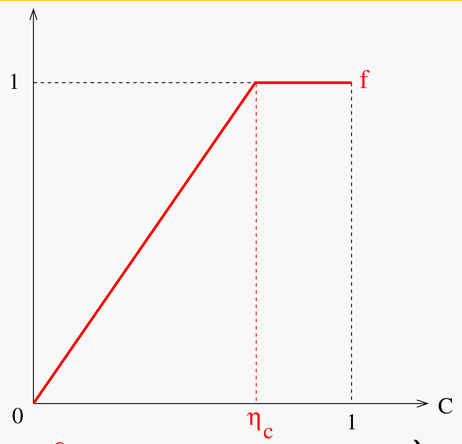

Crops suffer from water stress (resp. nitrogen stress) when K_S is not maximum (resp. f is not maximum).

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- $$\max B(T) \text{ under minimal water consumption } \Longleftrightarrow \min_{u(\cdot)} \int_0^T u(\tau) d\tau$$
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CONSTRAINED OPTIMAL CONTROL PROBLEM

$$\min_{u(\cdot)} \int_0^T u(\tau)d\tau, \quad (S^u_{0,S_0}(t), N^u_{0,S_0,N_0}(t)) \in E := \{(S,N) \in [0,1] \times \mathbb{R}_+ \; ; \; S \geq S^\star, \; N \geq \eta_c S\}\,,$$
$$t \in [0,T]$$

PROPOSITION

Depending on the initial N_0, one of the four strategies below is optimal.

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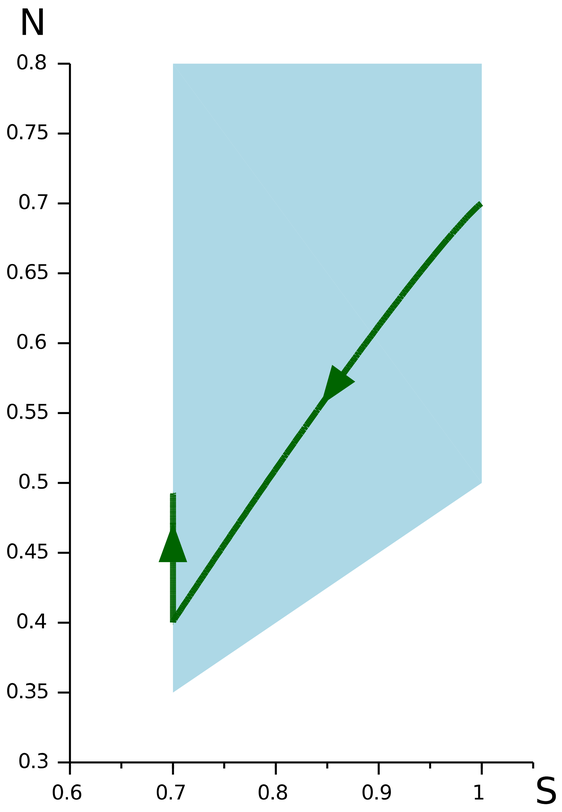

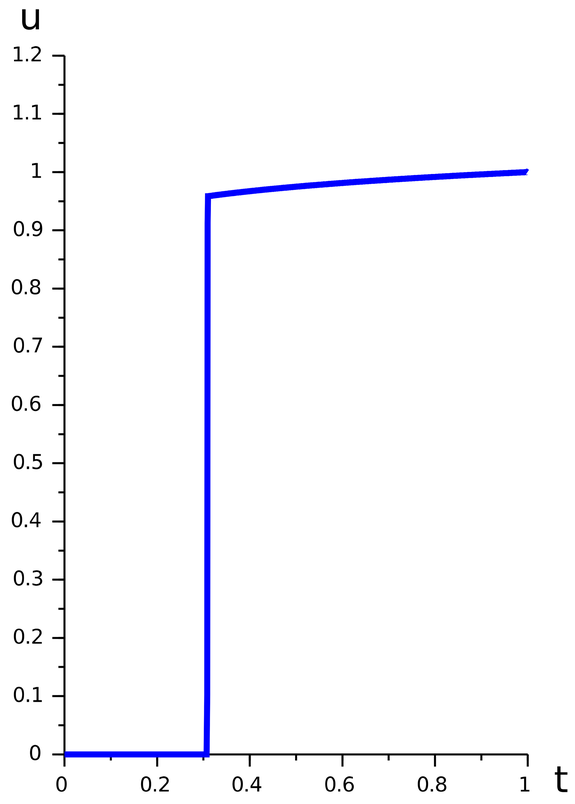

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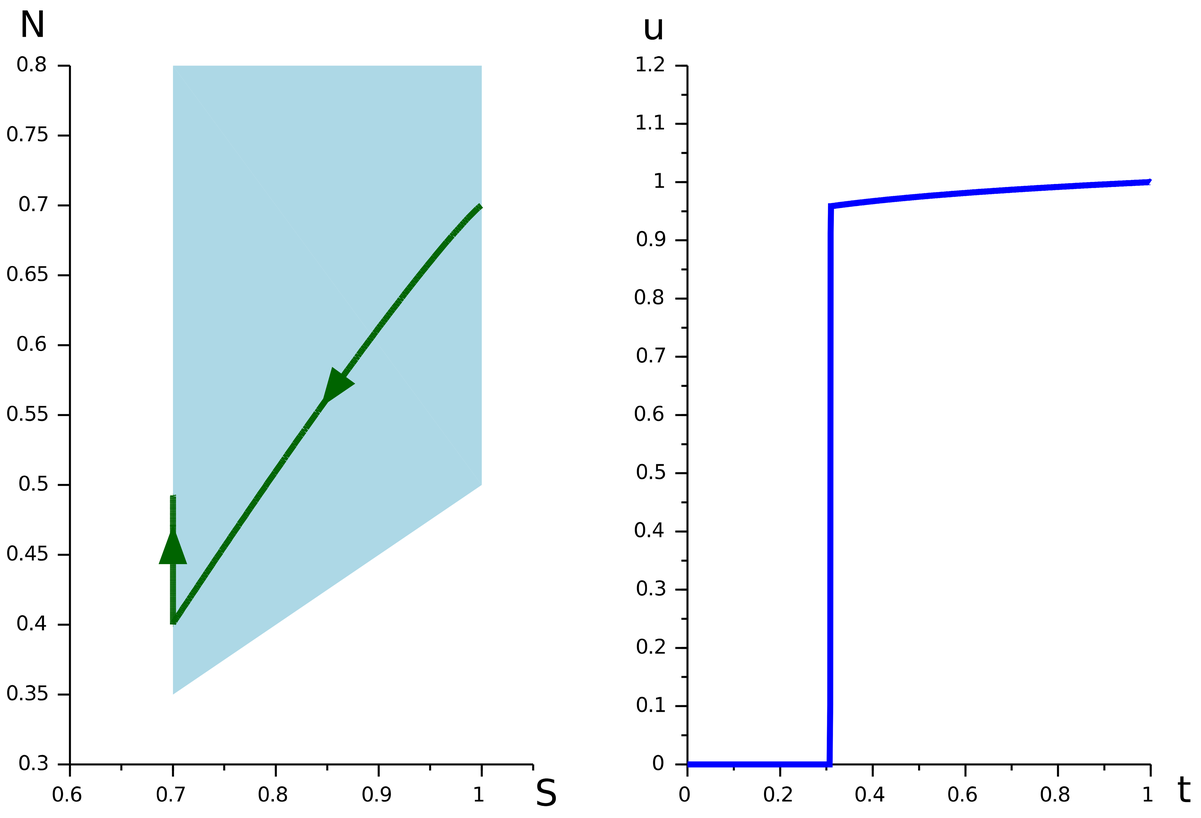
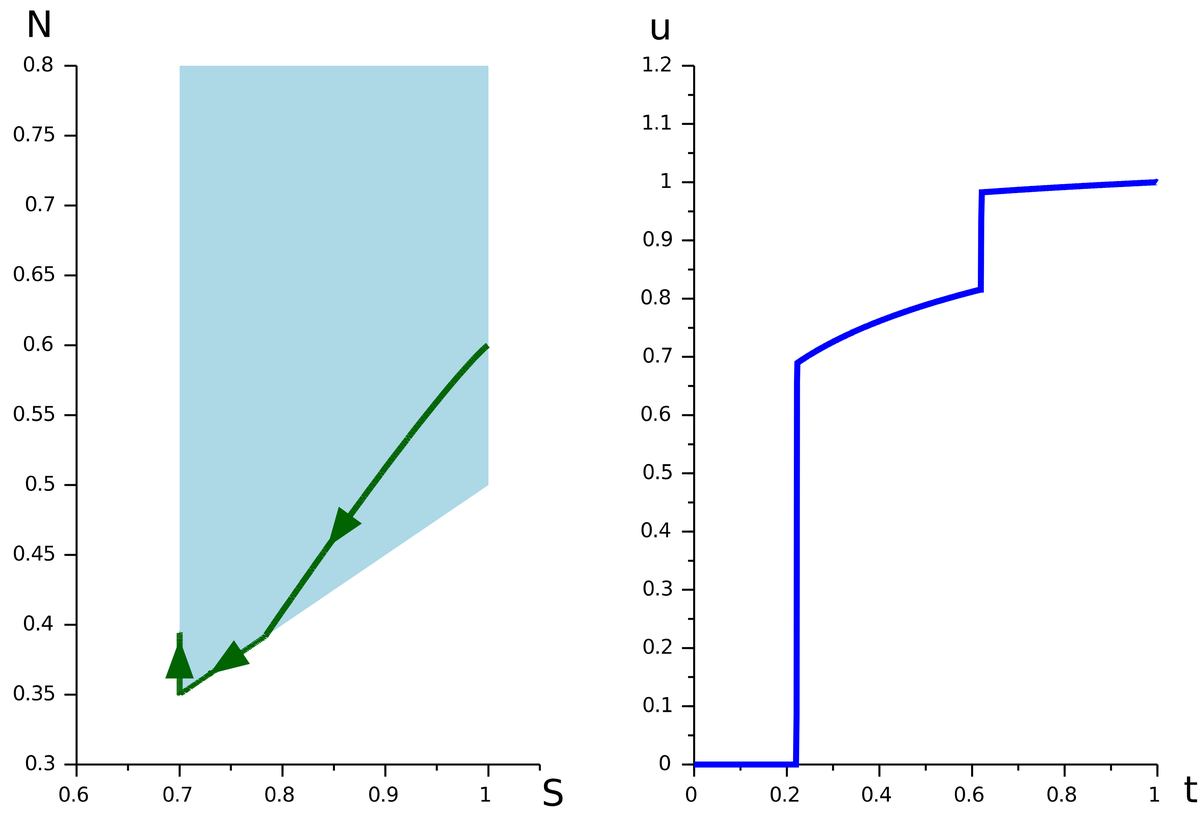


STRATEGY 2—> $N_0 \in [N_0^2, N_0^1]$



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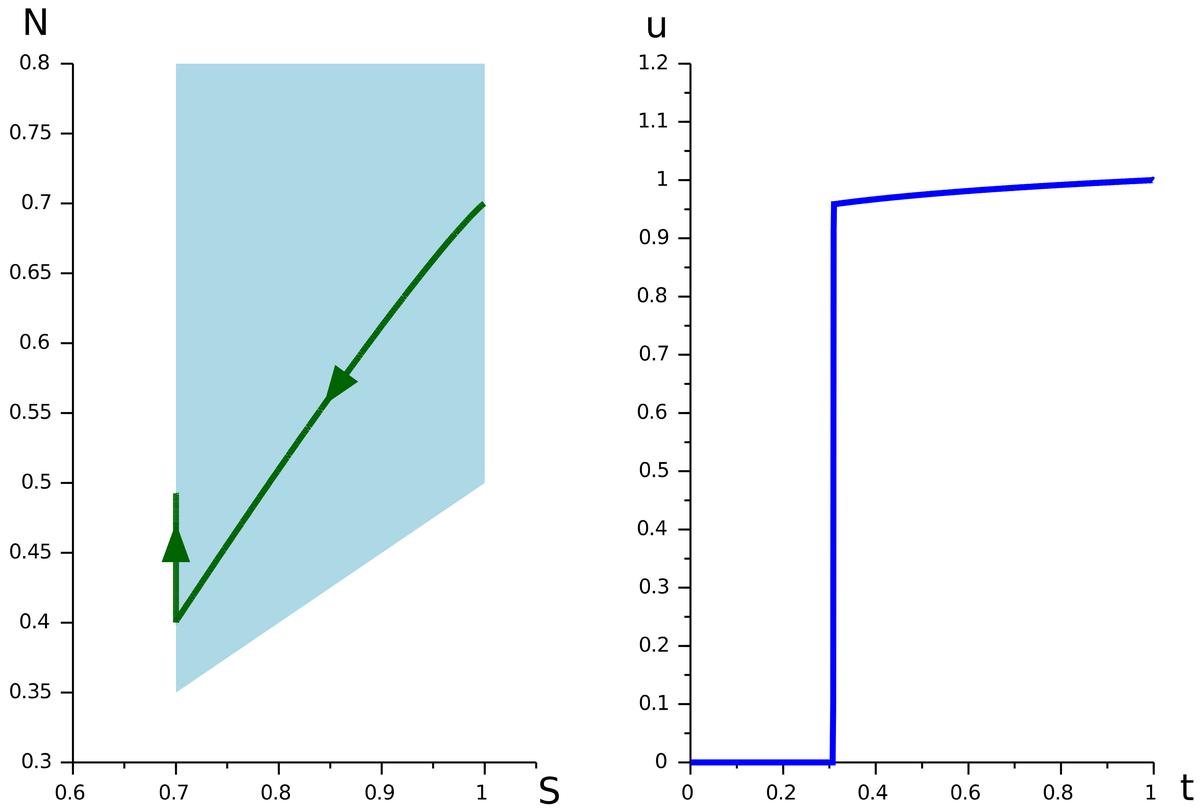

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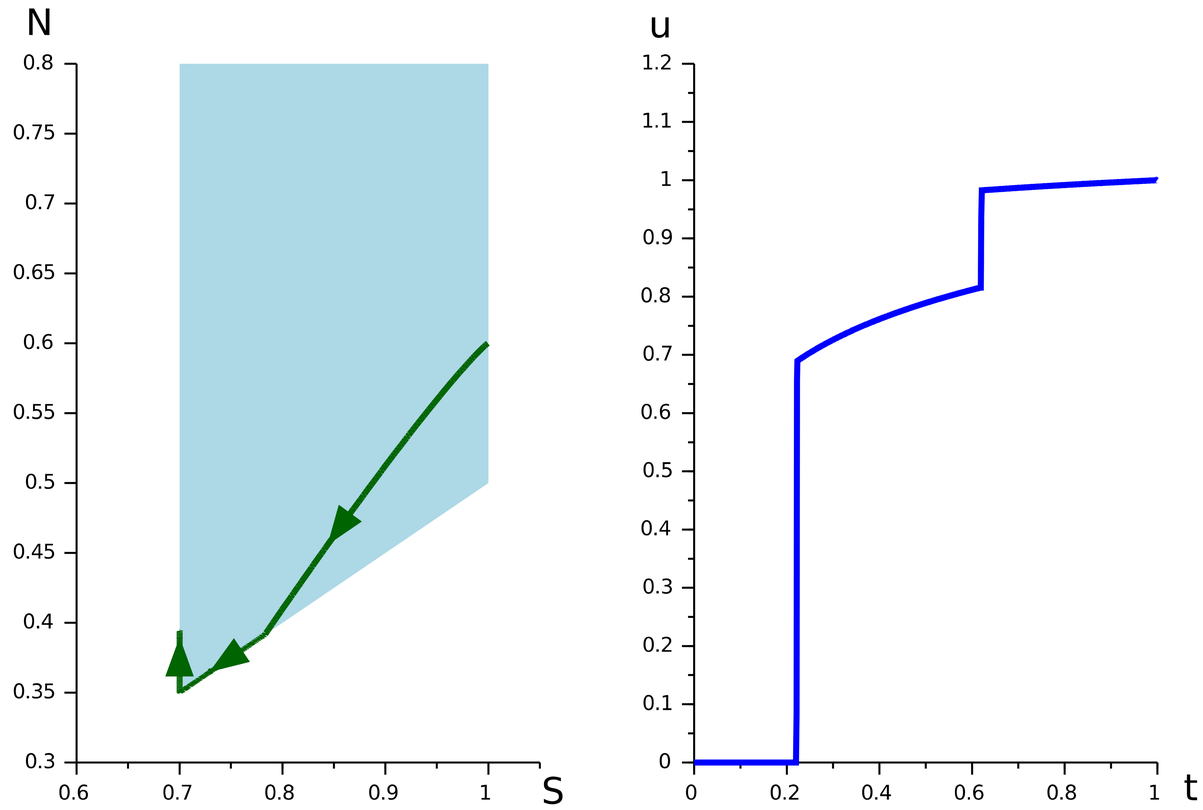
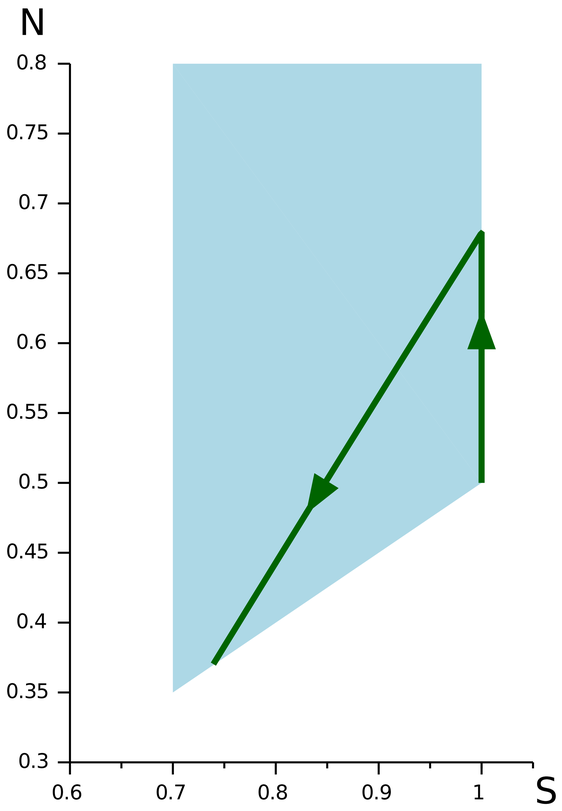
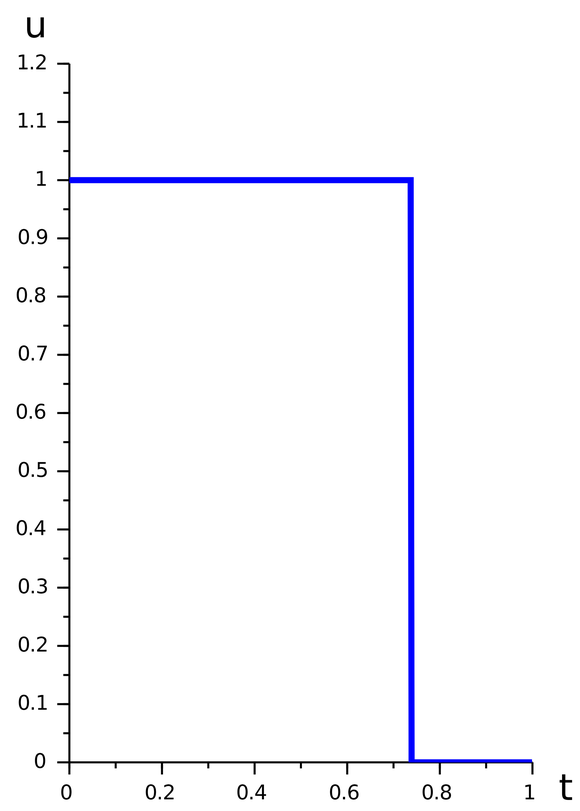


INTUITIVE STRAGIES

STRATEGY 3—>$N_0 \in [\eta_C, N_0^3]$





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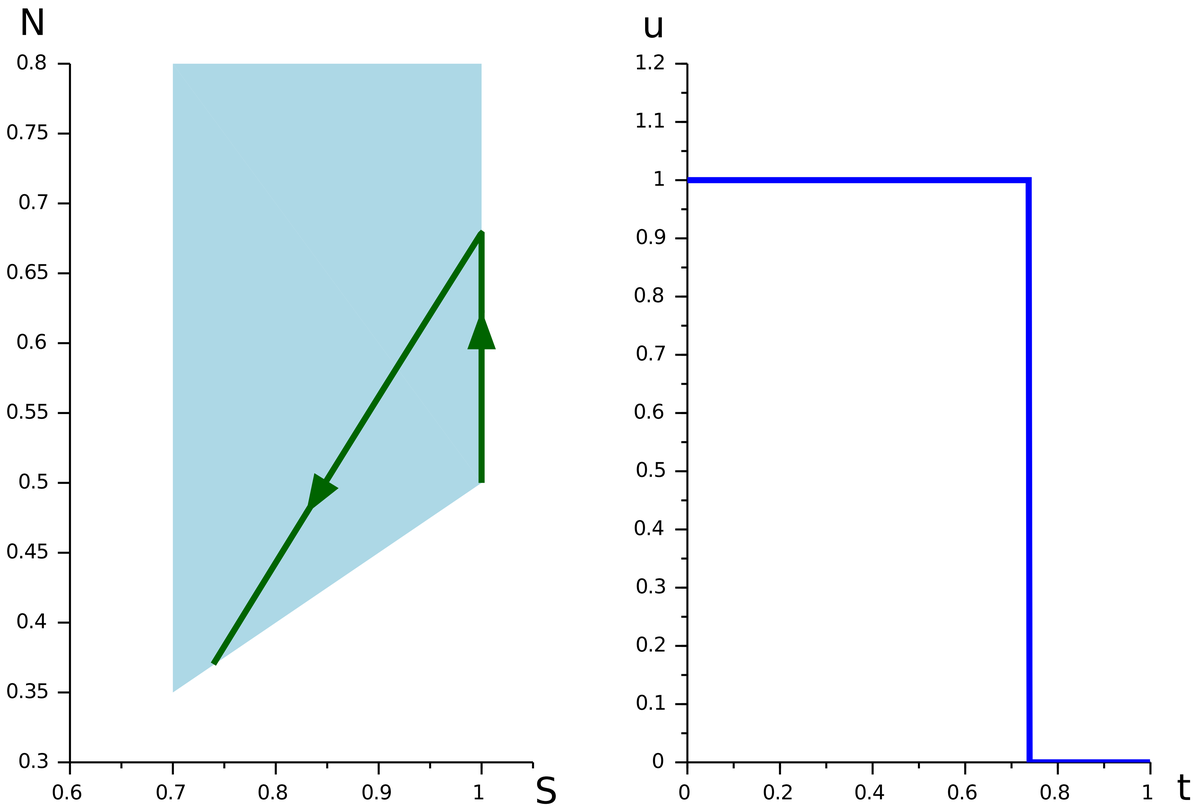

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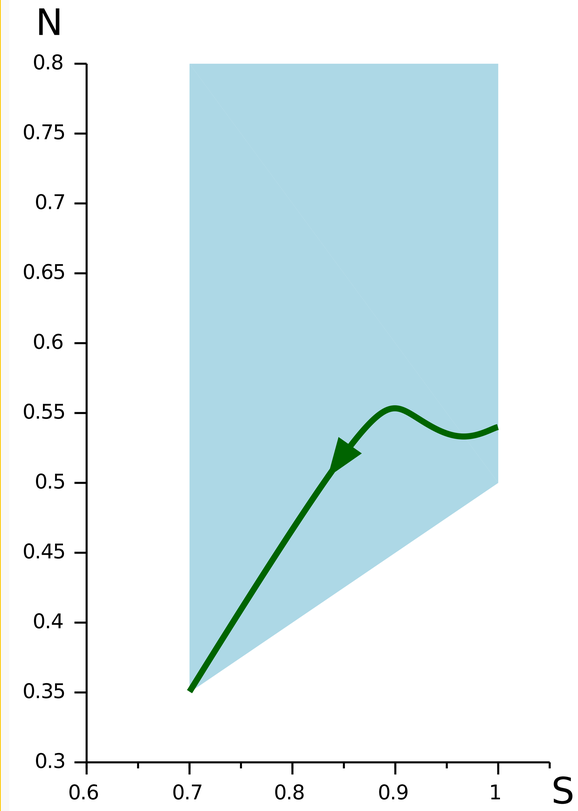

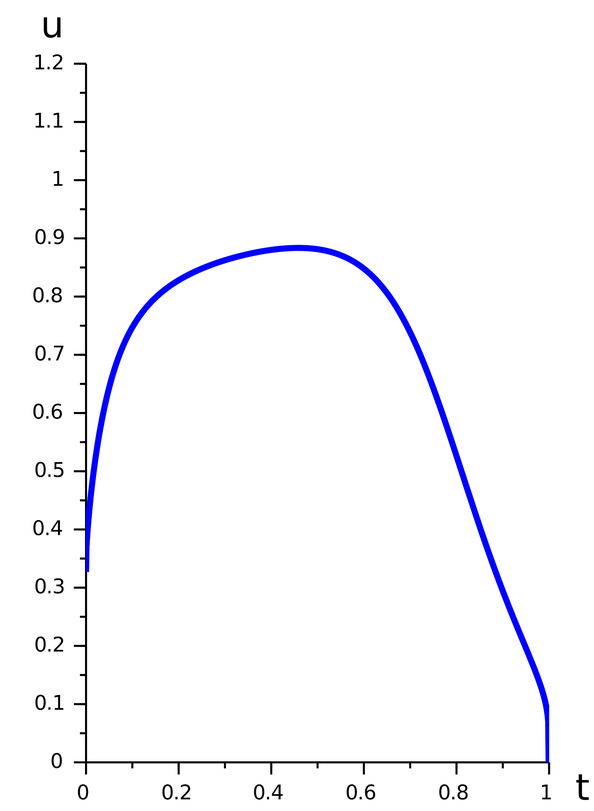

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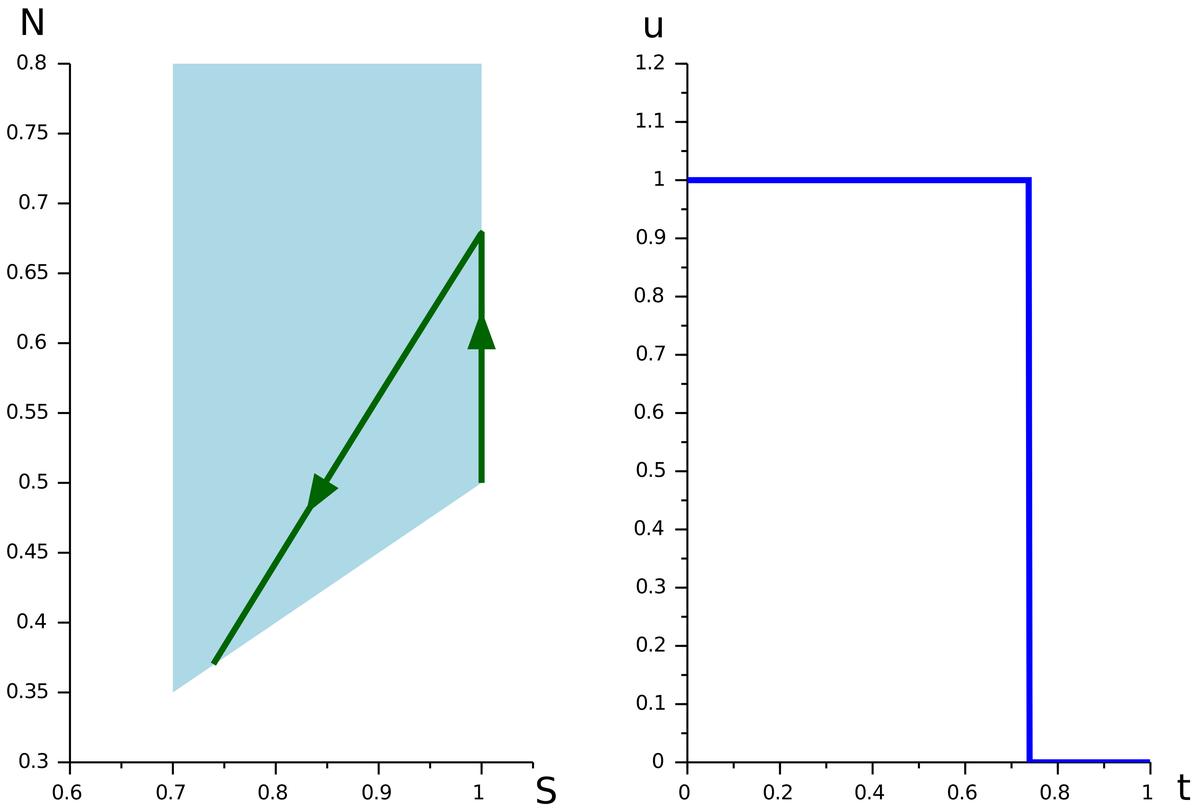

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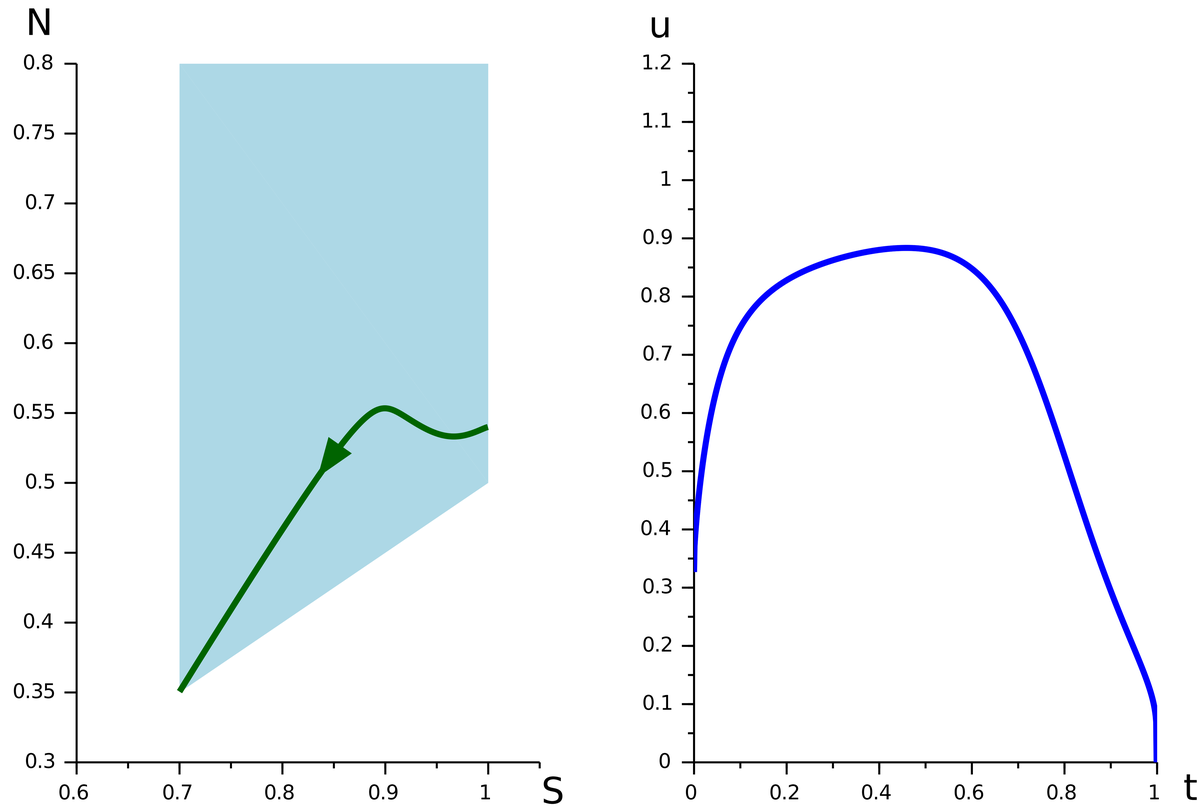

NON-INTUITIVE STRAGIES

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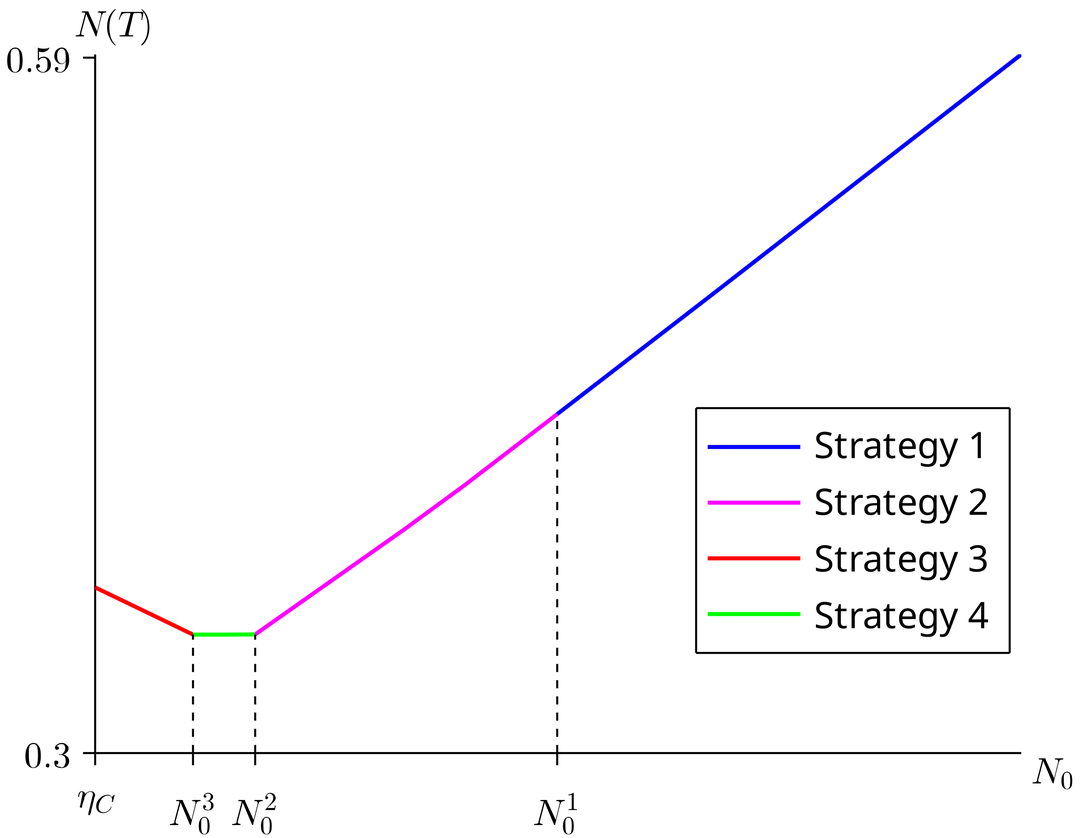

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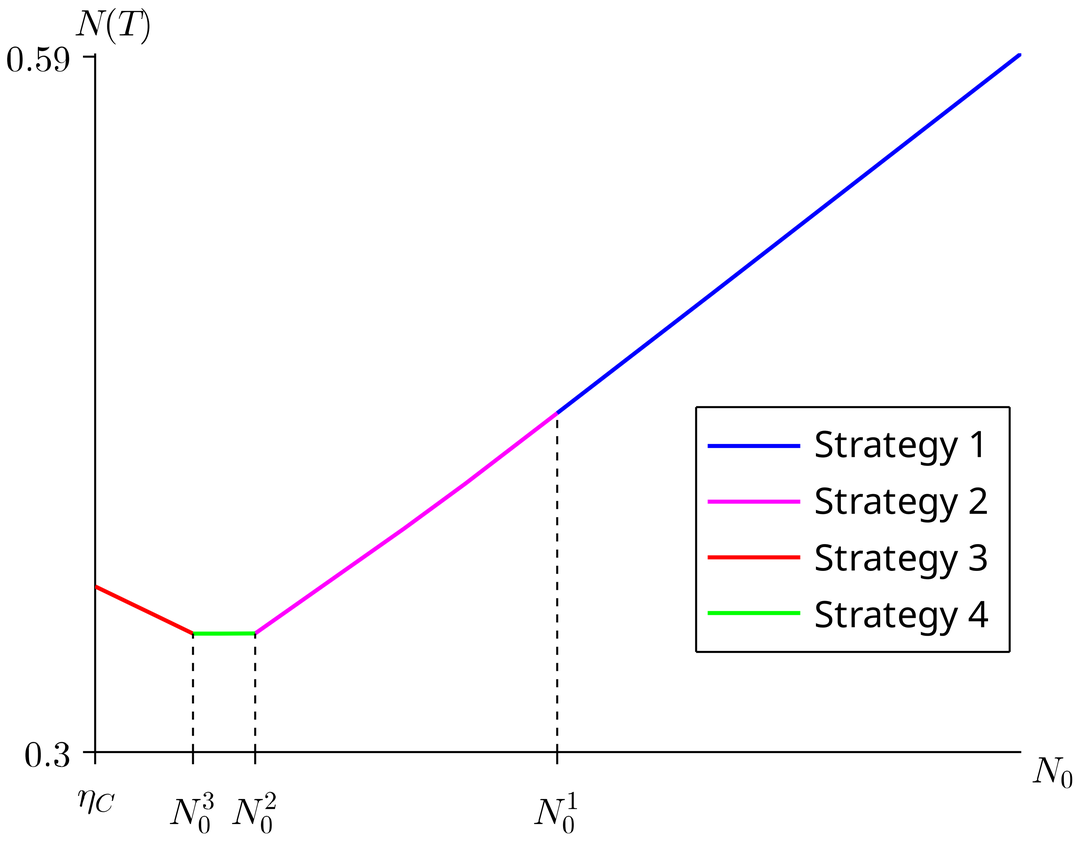
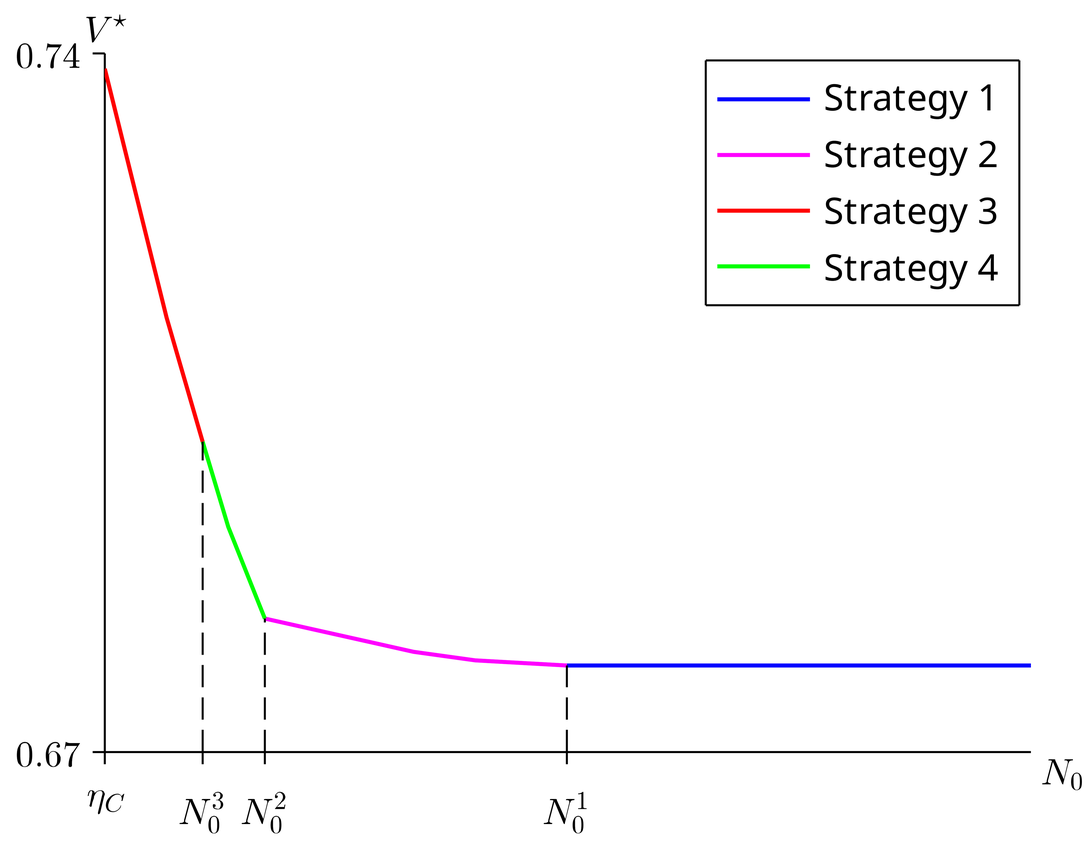
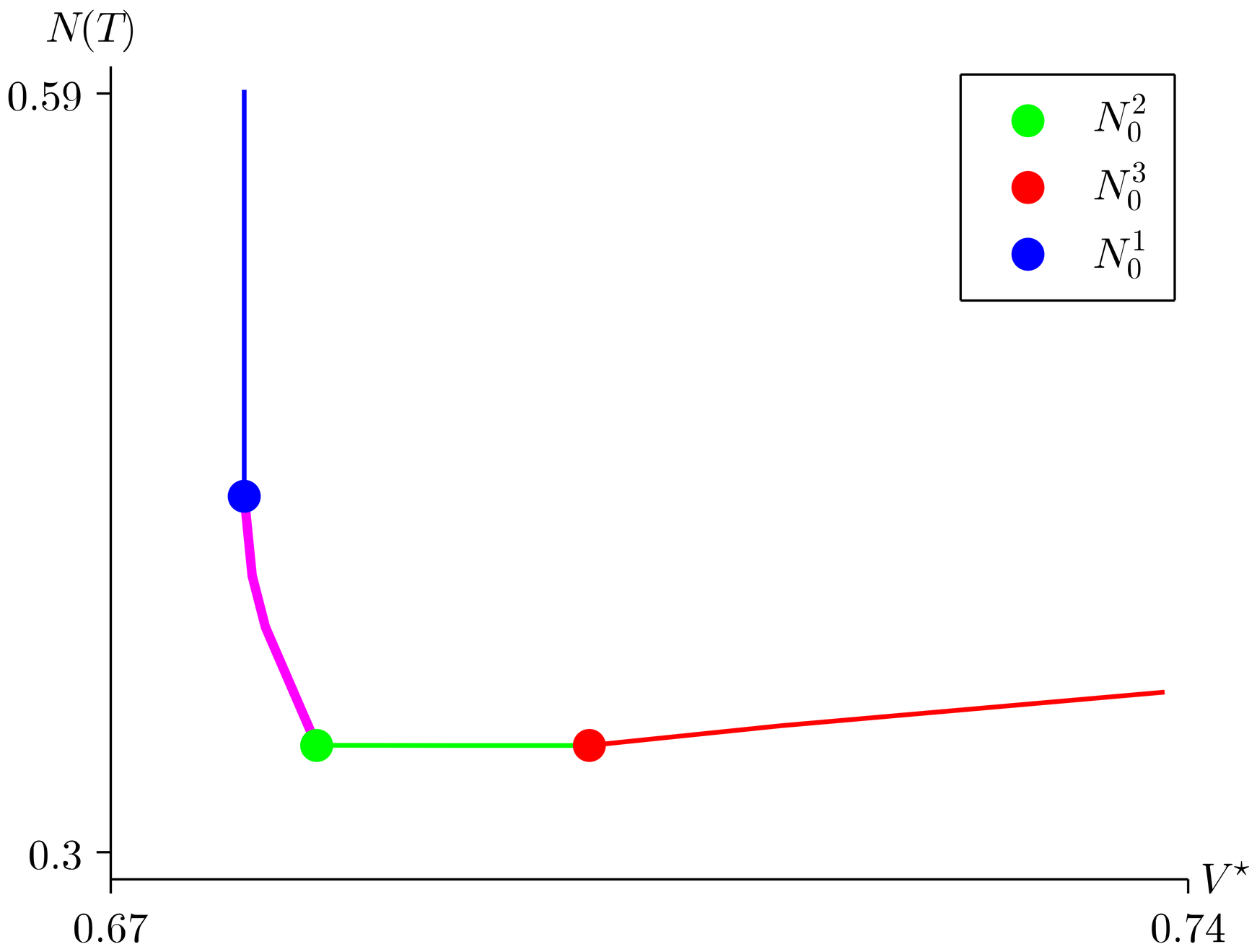


MINIMAL WATER CONSUMPTION



PARETO FRONT



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- If the initial nitrogen is high enough, wait for the right time to start irrigation.
- If the initial nitrogen is very low, irrigation must be anticipated from the start of the season.

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