Performance of a serial configuration of two chemostats.

Manel DALI YOUCEF
MISTEA, INRAE, Institut Agro, Univ Montpellier.

Alain RAPAPORT
MISTEA, INRAE, Institut Agro, Univ Montpellier.

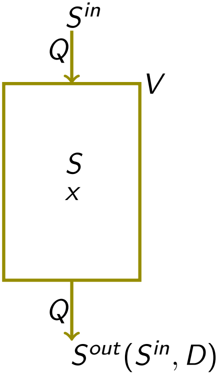
Tewfik SARI
ITAP, INRAE, Institut Agro, Univ Montpellier.

27 janvier 2020

LBE Modeling Thematic Day
Processes for REUSE

- Performance of a single chemostat
- Performance of two interconnected chemostats in serial.

Single chemostat



S^{out} : Output substrate concentration at steady state.
S^{in} : Input substrate concentration.
Q : Flow rate.
V : Total volume.
$D := Q/V$: The dilution rate.

S : Substrate concentration.
x : Biomass quantity.

$$\begin{cases} \dot{S} = D(S^{in} - S) - f(S)x \\ \dot{x} = -Dx + f(S)x. \end{cases}$$

- The function f is $\mathcal{C}^1$, with $f(0) = 0$ and $f'(S) > 0$ for all $S > 0$.
- The break-even concentration : $\lambda(D) := f^{-1}(D)$, $D \in [0, m)$ and $m := \sup_{S>0} f(S)$.

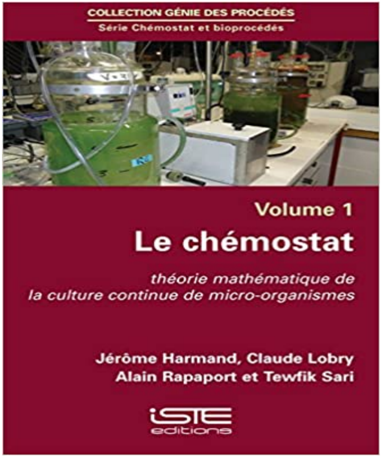
Unavoidable references

Hal L. Smith and Paul Waltman

Cambridge Studies in Mathematical Biology

The Theory of the Chemostat

Dynamics of Microbial Competition



COLLECTION GÉNIE DES PROCÉDÉS

Série Chémostat et bioprocédés

Volume 1

Le chémostat

théorie mathématique de la culture continue de micro-organismes

Jérôme Harmand, Claude Lobry

Alain Rapaport et Tewfik Sari

iSTE editions

Biomass productivity P and biogas flow rate G of a single chemostat

The productivity of the biomass

$P(S^{in}, D) := Qx^* = VD(S^{in} - \lambda(D))$ with $x^* = S^{in} - \lambda(D)$.

The biogas flow rate

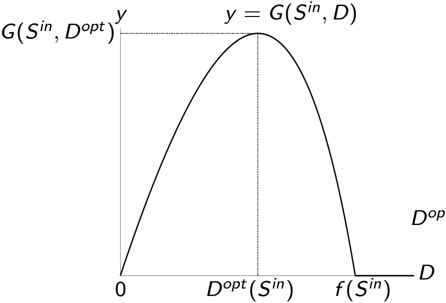
$G(S^{in}, D) := Vx^* f(S^*) = VD(S^{in} - \lambda(D))$ with $S^* = \lambda(D)$.

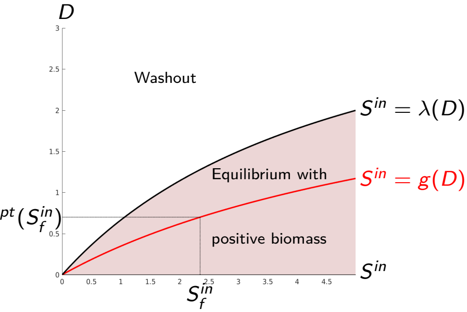
$$P(S^{in}, D) = G(S^{in}, D)$$

The dilution rate $D^{opt}(S^{in}) := \underset{0 \leq D \leq f(S^{in})}{\operatorname{argmax}} G(S^{in}, D)$ is the solution of $S^{in} = g(D)$ where

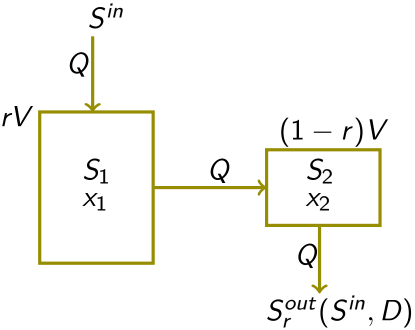
$$g(D) := \lambda(D) + \frac{D}{f'(\lambda(D))}.$$

The maximal biomass productivity of the single chemostat





Two interconnected chemostats



S_r^{out} : Output substrate concentration at steady state.
S^{in} : Input substrate concentration.
Q : Flow rate.
$r \in [0,1]$. V : Total volume.
$D := Q/V$: The dilution rate.

S_i : Substrate concentration of the tank $i = 1, 2$.
x_i : Biomass quantity of the tank $i = 1, 2$.

$$\begin{cases} \dot{S}_1 & = & \frac{D}{r}(S^{in} - S_1) - f(S_1)x_1 \\ \dot{x}_1 & = & -\frac{D}{r}x_1 + f(S_1)x_1 \\ \dot{S}_2 & = & \frac{D}{1-r}(S_1 - S_2) - f(S_2)x_2 \\ \dot{x}_2 & = & \frac{D}{1-r}(x_1 - x_2) + f(S_2)x_2. \end{cases}$$

- $E_0 = (S^{in}, 0, S^{in}, 0)$, • $E_1 = (S^{in}, 0, \overline{S}_2, \overline{x}_2)$, • $E_2 = (S_1^*, x_1^*, S_2^*, x_2^*)$.

The productivity of the biomass

$$P_r(S^{in}, D) := Qx_r^{out}$$

with x_r^{out} the output biomass of the second tank at steady state.

The productivity of the biomass corresponding to E_1

$$P_{r1}(S^{in}, D) := VD\overline{x}_2 = VD\left(S^{in} - \lambda\left(\frac{D}{1-r}\right)\right)$$

with $\overline{x}_2 = S^{in} - \lambda\left(\frac{D}{1-r}\right)$.

The productivity of the biomass corresponding to E_2

$$P_{r2}(S^{in}, D) := VDx_2^* = VD(S^{in} - S_2^*(S^{in}, D, r))$$

with $x_2^* = S^{in} - S_2^*(S^{in}, D, r)$.

The biogas flow rate

$$G_r(S^{in}, D) := \sum_{i=1}^{2} V_i x_i^* f(S_i^*)$$

with x_i^* and S_i^* the biomass and the substrate concentrations, at steady state, of the tank i , $i = 1, 2$.

The biogas flow rate corresponding to E_1

$$G_{r1}(S^{in}, D) := V_2 \overline{x}_2 f(\overline{S}_2) = VD\left(S^{in} - \lambda\left(\frac{D}{1-r}\right)\right)$$

with $V_2 = (1-r)V$, $\overline{x}_2 = S^{in} - \lambda(D/(1-r))$ and $\overline{S}_2 = \lambda(D/(1-r))$.

The biogas flow rate corresponding to E_2

$$\begin{aligned} G_{r2}(S^{in}, D) &:= V_1 x_1^* f(S_1^*) + V_2 x_2^* f(S_2^*) \\ &= VD\left(S^{in} - S_1^*\right) + V(1-r) f(S_2^*)(S^{in} - S_2^*) \\ &= VD(S^{in} - S_2^*(S^{in}, D, r)). \end{aligned}$$

with $V_1 = rV$, $x_1^* = S^{in} - S_1^*$, $S_1^* = \lambda\left(\frac{D}{r}\right)$, $V_2 = (1-r)V$ and $x_2^* = S^{in} - S_2^*$.

Biomass productivity and biogas flow rate of a serial configuration

Results

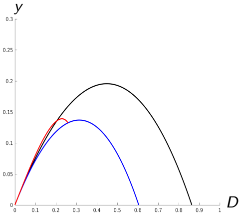
- $P_{r1}(S^{in}, D) = G_{r1}(S^{in}, D)$.
- $P_{r2}(S^{in}, D) = G_{r2}(S^{in}, D)$.

Results

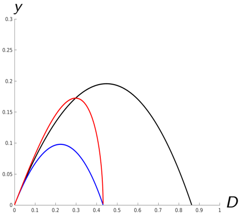
- $P_{r1}(S^{in}, D) < P(S^{in}, D)$ (always).
- $P_{r2}(S^{in}, D) > P(S^{in}, D) \iff S^{in} > g_r(D)$

with $g_r(D) := \lambda(D) + \frac{1}{1-r}\left(\lambda\left(\frac{D}{r}\right) - \lambda(D)\right)$.

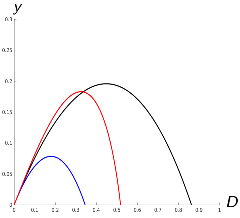
- $y = P(S^{in}, D)$
- $y = P_{r1}(S^{in}, D)$
- $y = P_{r2}(S^{in}, D)$



$0 < r < 1/2$

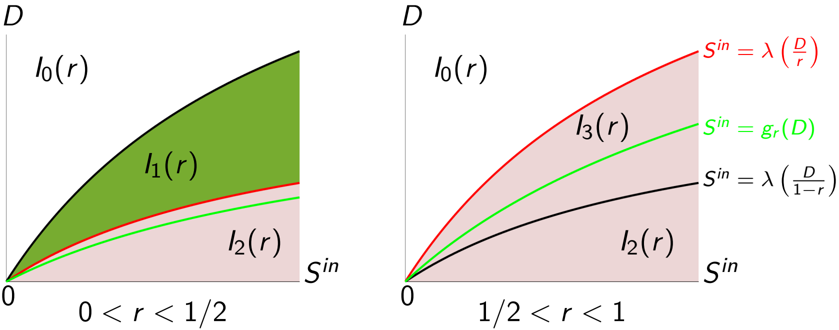


$r = 1/2$

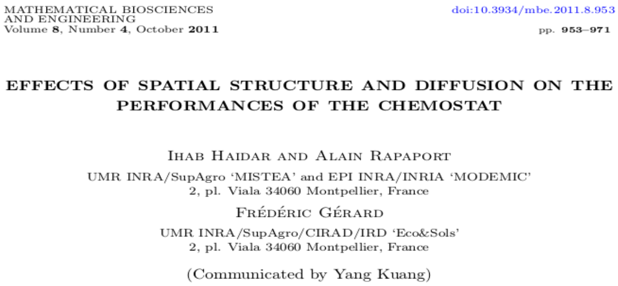


$1/2 < r < 1$

Biomass productivity and biogas flow rate of a serial configuration



	$I_0(r)$	$I_1(r)$	$I_2(r)$	$I_3(r)$
E_0	GAS	U	U	U
E_1		GAS	U	
E_2			GAS	GAS



MATHEMATICAL BIOSCIENCES
AND ENGINEERING
Volume **8**, Number **4**, October **2011**

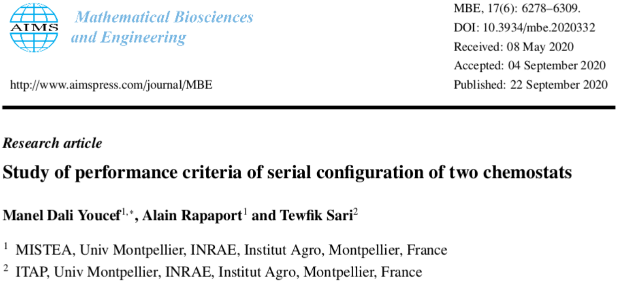
doi:10.3934/mbe.2011.8.953
pp. **953–971**

EFFECTS OF SPATIAL STRUCTURE AND DIFFUSION ON THE PERFORMANCES OF THE CHEMOSTAT

Ihab Haidar and Alain Rapaport
UMR INRA/SupAgro 'MISTEA' and EPI INRA/INRIA 'MODEMIC'
2, pl. Viala 34060 Montpellier, France

Frédéric Gérard
UMR INRA/SupAgro/CIRAD/IRD 'Eco&Sols'
2, pl. Viala 34060 Montpellier, France

(Communicated by Yang Kuang)



Mathematical Biosciences and Engineering

http://www.aimspress.com/journal/MBE

MBE, 17(6): 6278–6309.
DOI: 10.3934/mbe.2020332
Received: 08 May 2020
Accepted: 04 September 2020
Published: 22 September 2020

Research article

Study of performance criteria of serial configuration of two chemostats

Manel Dali Youcef[1,*], **Alain Rapaport**[1] **and Tewfik Sari**[2]

[1] MISTEA, Univ Montpellier, INRAE, Institut Agro, Montpellier, France
[2] ITAP, Univ Montpellier, INRAE, Institut Agro, Montpellier, France

- Two interconnected chemostats where the growth function is non-monotonic.
- Two interconnected chemostats with mortality rate and with a monotonic growth function.
- Two interconnected chemostats with competition and with a monotonic growth function.

for your attention.